

## MATHEMATICS

Time :  $2\frac{1}{2}$  hours

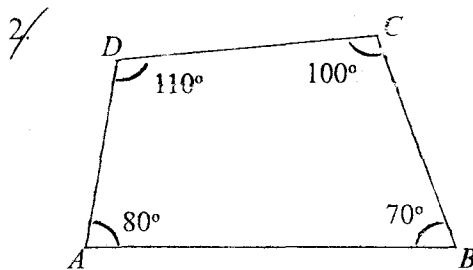
Total Score: 80

**Instructions:**

- Read the questions carefully, understand each question and then answer the questions.
- Give explanations wherever necessary.
- If there is an OR between any two questions, you may answer only one among them.
- 15 minutes will be given at the beginning as cool off time. This time may be utilised to read and understand the questions.
- Simplifications using  $\pi$ , etc., with their approximate values are not required if not specified in the question.

SCORE

1. ✓ Write down an arithmetic sequence with common difference 6. What is the  $n^{\text{th}}$  term of this sequence? [2]



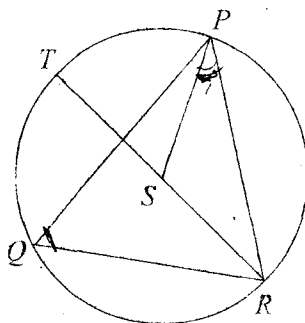
- ✓ In the quadrilateral  $ABCD$  shown in the figure,  $\angle A = 80^\circ$ ,  $\angle B = 70^\circ$ ,  $\angle C = 100^\circ$  and  $\angle D = 110^\circ$ . Is this quadrilateral  $ABCD$  a cyclic quadrilateral? [2]

Is the circle with diameter  $BD$  passes through  $A$ ? Why?

3. ✓ The two opposite vertices of a rectangle whose sides parallel to axes are  $(3, 2)$  and  $(7, -4)$ . Write the co-ordinates of the other two vertices of the rectangle. [2]

4.  $15, 33, 51, \dots$  is an arithmetic sequence. Prove that, if 1 is added to the sum of the continuous terms of this sequence, it will be a perfect square. [3]

5.



In the figure  $S$  is the centre of the circumcircle of  $\Delta PQR$ .  $RT$  is a diameter of the circle. [3]

Prove that  $\angle PQR + \angle RPS = 90^\circ$ .

6. A square pyramid has the base edge 14 centimetre and the lateral edge 25 centimetre. Find the height and slant height of this pyramid. [3]

7. In  $\triangle ABC$ ,  $\angle A = 80^\circ$ ,  $AB = 15$  centimetre and  $AC = 8$  centimetre. [3]

(a) What is the perpendicular distance from  $C$  to  $AB$ ?

(b) Calculate the area of the triangle.

$$(\sin 80^\circ = 0.98, \quad \cos 80^\circ = 0.17, \quad \tan 80^\circ = 5.6)$$

8. The area of a rectangle with length 6 centimetre more than the breadth, is 135 square centimetre. Find the length and breadth of the rectangle. [3]

OR

The perimeter of a rectangle is 40 centimetre. Can its area be 120 square centimetre? Justify your answer.

9. A box contains 3 white balls and 7 black balls. Another box contains 4 white balls and 6 black balls. If we choose one ball each from the two boxes without looking in to the boxes, [3]

(a) what is the probability of getting two balls with the same colour?

(b) what is the probability of getting two balls with different colours?

10. Draw the  $x$ -axis and  $y$ -axis in the answer sheet. [3]

Mark the points whose co-ordinates are  $(5, 3)$ ,  $(-3, 4)$ ,  $(2, 0)$  and  $(-1, -3)$ .

11. When the polynomial  $2x^3 + kx^2 + 17x - 2$  is divided by  $(x - 2)$  and by  $(x - 3)$  we get the same remainder. [3]

(a) Find the value of  $k$ .

(b) Check whether  $(2x - 1)$  is a factor of this polynomial.

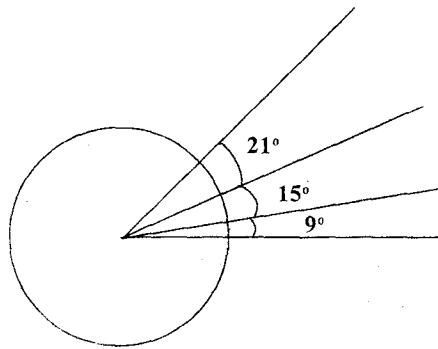
12. Can  $A(-1, 5)$ ,  $B(2, 1)$ ,  $C(7, -11)$  be the vertices of a triangle? Justify your answer. [3]

13. The electricity consumption of 50 houses of a colony is shown in the table below: [3]

<i>Electricity consumption (in units)</i>	<i>Number of houses</i>
0 – 20	2
20 – 40	6
40 – 60	8
60 – 80	10
80 – 100	12
100 – 120	7
120 – 140	5

Find the mean consumption.

14. The base angles of an isosceles triangle are  $50^\circ$  each. The radius of the circumcircle of this triangle is 3.5 centimetres. Construct the triangle. [4]
15. Thin metal rods of length 10 centimetre each is placed one end joined at a point to get a circular pattern. [4]



In this pattern the angle between the near two rods is increased by  $6^\circ$ . The smallest angle is  $9^\circ$ . Find the total length of the rods required to make this pattern.

OR

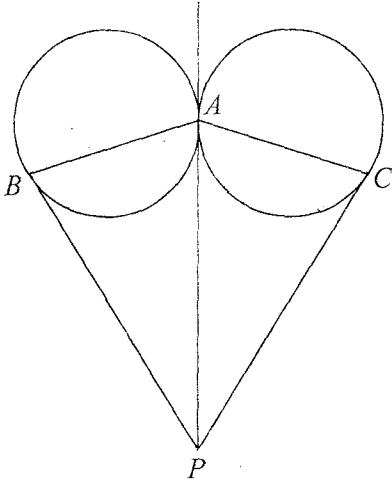
The expense for building the first metre of a clock tower is 4500 rupees, 500 rupees more for the next metre and so on. The total height of the tower is 15 metres.

- (a) How much more amount is needed to build the last metre of the tower than the first metre?
- (b) What will be the total expense for building the tower?

16. In  $\triangle ABC$ ,  $AB = 7$  centimetre,  $\angle B = 65^\circ$ ,  $AC = 6.5$  centimetre. Construct  $\triangle ABC$  and its incircle. Write the radius of the incircle. [4]

17. Write  $2x^2 - 5x - 3$  as the product of two first degree polynomials. [4]

18. [4]



In the figure two circles touch at  $A$ .  $P$  is a point on the tangent through  $A$ .  $PB$  and  $PC$  are tangents and  $PA$  is the bisector of  $\angle BPC$ . Then prove that  $AB = AC$ .

19. Scores of the students of a class in Mathematics examination is given in the table below: [4]

Score	Number of students
0 – 10	1
10 – 20	2
20 – 30	5
30 – 40	8
40 – 50	10
50 – 60	7
60 – 70	5
70 – 80	2

Find the median of scores.

20.  $A$  and  $B$  are two cities 300 kilometres apart. A car travels from  $A$  in uniform speed and reaches  $B$ . If the speed of the car is increased by 5 kilometres/hour it could have reached at  $B$  2 hours early [5]

- Taking the speed of the car as  $x$ , form a second degree equation.
- Find the speed of the car.

OR

The distance between the opposite vertices of a rectangular shaped plot is 26 metres. The length of this plot is 4 metres more than twice its breadth.

- If the breadth is  $x$ , what is the length?
- What is the cost of wire fencing around the plot at the rate of 80 rupees per metre?

21. In  $\triangle ABC$ ,  $BC = 10$  centimetres,  $\angle ABC = 65^\circ$  and  $\angle ACB = 75^\circ$ . [5]

- What is the diameter of the circumcircle of  $\triangle ABC$
- Using this diameter find the length of the other two sides of the triangle.

$$\begin{array}{lll} (\sin 65^\circ = 0.91 & \sin 75^\circ = 0.97 & \sin 40^\circ = 0.64 \\ \cos 65^\circ = 0.42 & \cos 75^\circ = 0.26 & \cos 40^\circ = 0.76 ) \end{array}$$

22. A solid cone made of metal has slant height 10 centimetre and height 8 centimetre. On the base of this cone a solid hemisphere with the same metal is attached. [5]

- Find the common radius of this solid.
- If we melt this solid and recast in to small solid spheres of radius 6 millimetres, how many complete spheres can be made?

OR

A metal solid is in the shape of hemispheres of same radii attached to the end faces of a cylinder. The total length of the solid is 10 metres and the common radius is 3 metres.

- What is the volume of this solid?
- What is the cost of painting the surface of this solid at the rate of 250 rupees per square metre?

23.  $3x + 2y - 8 = 0$  and  $2x - 3y - 1 = 0$  are equations of two lines. [5]

- Find in which of the above lines  $A(4, -2)$  and  $B(5, 3)$  belongs.
- If  $C$  is a point on the above two lines, write the coordinates of  $C$ .
- Find the product of the slopes of the lines  $CA$  and  $CB$ .